A Theory of Three-way Decisions (三支决策)

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May 23, 2014

感谢

- 西安工程大学科技大讲堂提供这次报告机会.
- 西安工程大学理学院的邀请.
- 西安工程大学科技处、研究生部的组织安排.
- Professor 贺兴时
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- Dr. 李小南, 西安电子科技大学
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- Dr. 祁建军, 西安电子科技大学
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《论语·先进》

“子贡问：‘师与商也孰贤？’子曰：‘师也过，商也不及。’曰：‘然则师愈与？’子曰：‘**过犹不及**。’”

春秋时期，孔子的学生子贡问孔子他的同学子张和子夏哪个更贤明一些。孔子说子张常常超过周礼的要求，子夏则常常达不到周礼的要求。子贡又问，子张能超过是不是好一些，孔子回答说超过和达不到的效果是一样的。

`http://baike.sogou.com/v112495.htm`

Three-way decisions: 不及, 中庸, 过

From two-way decisions to three-way decisions: Spatial (空间)

- top, middle, bottom,
- 上, 中/不上不下, 下
- front, middle, back
- 前, 中/不前不后, 后
- left, center, right
- 左, 中/不左不右, 右

From two-way decisions to three-way decisions: Temporal (时间)

- yesterday, today, tomorrow
- 昨天, 今天, 明天
- past, present, future
- 过去, 现在, 将来

From two-way decisions to three-way decisions: Size and volume (尺寸和体积)

- long, medium, short
- 长，不长不短，短
- high, medium, low
- 高, 不高不低, 低
- large, medium, small
- 大, 不大不小, 小

From two-way decisions to three-way decisions: Attitude (态度)

- positive, neutral, negative
- 正，中性，负
- accept, non-commitment, reject
- 接受，不承诺，拒绝

From two-way decisions to three-way decisions: Evaluation (评价)

- yes/right, maybe, no/wrong
- 是/对, 可能/不明确, 非/否/错
- upper/top, middle, lower/bottom
- 高, 不高不低, 低
- good, so-so, bad
- 好，不好不坏，坏

Why?

- Cognitive basis and advantages
  - Humans tend to classify and category the world.
  - Humans have a limited information processing capability.

Organization and categorization

- Pinker S (1997) How the Mind Works. WW Norton & Company, New York.
  - Humans tend to organize.
  - Categorization is essential to mental life. capability.
  - Possible results of such organizations are some types of structures.
- **In three-way decisions, we have triparition**.

Limited human information processing capability

- G.A. Millller (1956) The magical number seven, plus or minus two: Some limits on our capacity for processing information, Psychological Review 101, 343-352.
- N. Cowan (2001) The magical number 4 in short-term memory: A reconsideration of mental storage capacity, Behavioral and Brain Sciences 24, 87-114.
- **The choice of three is appropriate**.

Main Ideas

- 三支决策受到Rough集启发
- 三支决策比Rough集更具有普适性

- Rough集可以产生三个区域
- 三个区域不一定由Rough集获得

- **应跳出粗糙集的圈子，同其他学科更紧密地联系起来，走出自己独特的路子.**

A brief history of development (My contributions)

- Yao, Y.Y. Three-way decision: an interpretation of rules in rough set theory, RSKT 2009, LNAI 5589, pp. 642-649, 2009.
- Yao, Y.Y. Three-way decisions with probabilistic rough sets, Information Sciences, Vol. 180, No. 3, pp. 341-353, 2010.
- Yao, Y.Y. The superiority of three-way decisions in probabilistic rough set models, Information Sciences, Vol. 181, No. 6, 1080-1096, 2011.
- Yao, Y.Y., An Outline of a Theory of Three-way Decisions. In: Yao, J., Yang, Y., RSCTC 2012. LNCS (LNAI), vol. 7413, pp. 1-17. Springer, Heidelberg (2012).
- Yao, Y.Y., Granular computing and sequential three-way decisions. In: RSKT 2013. LNAI 8171, pp. 16-27.
- 姚一豫, 三支决策 , 贾修一, 商琳, 周献中, 梁吉业, 苗夺谦, 王国胤, 李天瑞, 张燕平. 《三支决策理论与应用》, 南京大学出版社, 南京, 1-16, 2012.

A Few Related Studies (Examples):

- Liu, D., Li, T.R., Liang, D.C., Three-way government decision analysis with decision-theoretic rough sets, International Journal of Uncertainty, Fuzziness and Knowledge-based Systems 20 (2012) 119-132.
- Jia, X.Y., Zhang, K., Shang, L., Three-way decisions solution to filter spam email: An empirical study. RSCTC 2012. LNCS (LNAI), vol. 7413. Springer, Heidelberg (2012)
- Liu, D., Yao, Y.Y., Li, T.R., Three-way investment decisions with decision-theoretic rough sets, International Journal of Computational Intelligence Systems 4 (2011) 66-74
- Yu, H., Wang, Y., Three-way decisions method for overlapping clustering. RSCTC 2012. LNCS (LNAI), vol. 7413. Springer, Heidelberg (2012)

A Few Related Studies (Examples):

- 刘盾, 李天瑞. 三枝决策粗糙集// 李华雄, 周献中, 李天瑞, 王国胤, 苗夺谦, 姚一豫. 决策粗糙集理论及其研究进展. 北京: 科学出版社, 2011.
- 刘盾, 姚一豫, 李天瑞. 三枝决策粗糙集. 计算机科学, 2011, 38: 245 250.
- 贾修一, 李伟, 商 琳, 陈家骏. 一种自适应求三枝决策中决策阈值的算法. 电子学报, 2011, 39: 2520 2525.
- 贾修一, 商 琳. 一种求三支决策阈值的模拟退火算法, 2012, 手稿.
- 贾修一, 商琳, 陈家骏. 基于三支决策的属性约简// 中国人工智能进展, 2009: 193 198.
- 胡卉颖, 罗锦坤, 刘阿宁. 三枝决策粗糙集模型属性约简研究. 软件导刊, 2012, 11: 20 22.

A brief history of development

李华雄, 周献中, 李天瑞, 王国胤, 苗夺谦, 姚一豫. 决策粗糙集理论及其研究进展. 北京: 科学出版社, 2011.

A brief history of development

贾修一，商琳，周献中，梁吉业，苗夺谦，王国胤，李天瑞，张燕平. 三支决策理论与应用. 南京：南京大学出版社，2012.

A brief history of development

刘盾，李天瑞，苗夺谦，王国胤，梁吉业 （编著）．《三支决策与粒计算》，科学出版社， 北京，2013.

Special Session on Three-way Decisions and Probabilistic Rough Sets in JRS 2013

Organizers: Hong Yu, Bing Zhou, Dun Liu, Fan Min, Xiuyi Jia, and Huaxiong Li

Three-Way Decision Rough Sets

First International Workshop, 2013

The 2013 International Symposium on Three-way Decision and Granular Computing (ISTDGC2013)

August 12-14, 2013, Southwest Jiaotong University, Chengdu, China

Program at Glance

Monday, August 12, 2013	
10:00-19:00	Registration: **Mirror Lake Hotel, Southwest Jiaotong University**
Tuesday, August 13, 2013	
08:30-08:45	**Welcome and Opening Ceremony** (ROOM1)
08:50-09:50	Keynote 1: ***Yiyu Yao*** (ROOM1)
09:45-10: 30	**Photo & Coffee Break**
10: 30-11: 20	Keynote 2: ***Duoqian Miao*** (ROOM1)
11:20-12:10	Keynote 3: ***Yanping Zhang*** (ROOM1)
	Lunch (ROOM2)
14:30-15: 20	Keynote 4: ***Baoqing Hu*** (ROOM1)
15:20-16: 10	Keynote 5: ***Jiye Liang*** (ROOM1)
16:10-16: 30	**Coffee Break**
16:30-17: 20	Keynote 6: ***Lin Shang*** (ROOM1)
17:20-18:10	Keynote 7: ***Hong Yu*** (ROOM1)
18:10-19:30	**Dinner** (ROOM2)
19:30-21:30	**Seminar** (ROOM1)
Wednesday, August 14, 2013	
08:30-09:10	Keynote 8: ***Fan Min*** (ROOM1)
09:10-09:50	Keynote 9: ***Huaxiong Li*** (ROOM1)
09:50-10:10	**Coffee Break**
10:10-10:50	Keynote 10: ***Decui Liang*** (ROOM1)
10:50-11:30	Keynote 11: ***Xibei Yang*** (ROOM1)
11:30-12:10	Keynote 12: ***Hongkai Wang*** (ROOM1)
	Lunch (ROOM2)
14:30-15:20	Keynote 13: ***Dun Liu*** (ROOM1)
15:20-16:10	Keynote 14: ***Tianrui Li*** (ROOM1)

Second International Workshop, 2014

Call for Papers

The Second International Workshop on Three-way Decisions, Uncertainty, and Granular Computing

Tongji University, Shanghai, October 24-26, 2014

http://see.tongji.edu.cn/RSKT2014/

Three-way Decisions theory, formulated based on the notions of acceptance, rejection and no commitment, provides a meaningful semantics interpretation of the three regions in the rough set theory. It offers a new insight into Granular Computing with uncertainty. A broad scope of research on the Three-way Decisions will not only contribute to a better understanding of the essentials of rough set theory, but also effective applications in decision-making. In recent work, the three-way decisions have been successfully used for decision analysis under uncertainty, decision making, cluster analysis, information filtering, Web-based support systems, attribute reduction, multi-criteria classification and multi-view decision models.

Website of three-way decisions (You are welcome to join)

http://www2.cs.uregina.ca/~twd/

Homepage of Three-way Decisions

(三支决策主页)

QR Code of this homepage.
本主页二维码

Introduction to a Theory of Three-way Decisions (三支决策理论介绍)

News(相关新闻)

Researchers in Three-way Decisions (三支决策研究专家)

Publications (出版物)

- Books and journal special issues (出版书籍和专刊)
- List of papers by authors (论文列表—按作者序)

Conferences, Workshops, and Special Sessions (会议，研讨会和专题讨论)

JingTao Yao and Yan Zhang, A Scientometrics Study of Rough Sets in Three Decades, RSKT 2013

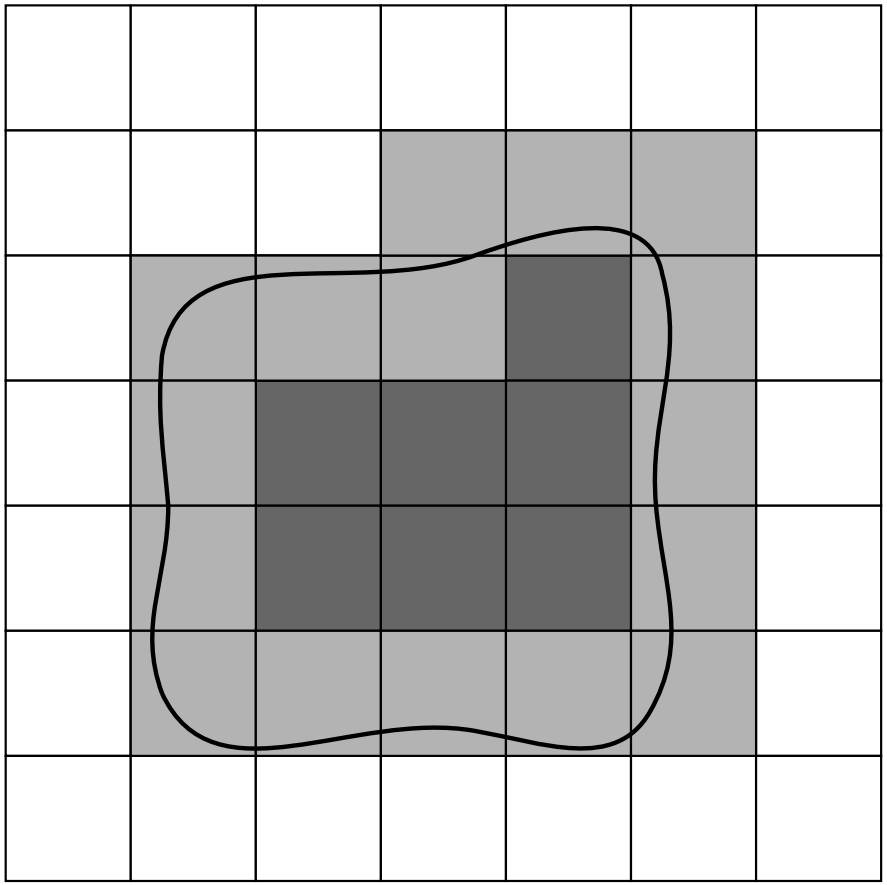
Table 6. Top 20 cited papers in recent 5 years

	Paper	Total Citations	Average per Year	Main Results
1	Feng+ 2008 [10]	91	15.17	Soft sets
2	Yao YY+2008 [73]	81	13.50	Reduction in DTRS
3	Yao YY 2008 [66]	76	12.67	Probabilistic rough sets
4	Hu+ 2008 [19]	75	12.50	App - feature subset selection
5	Zhu 2009 [74]	69	13.80	Generalized RS
6	Hu+ 2008 [20]	65	10.83	App - neighborhood classifier
7	Jensen+ 2009 [21]	64	12.80	App - feature selection
8	Wu 2008 [52]	60	10.00	Attribute reduction
9	Qian+ 2010 [41]	55	13.75	Reduction accelerator
10	Wang+2008 [49]	52	8.67	App - rule induction
11	Thangavel+ 2009 [48]	48	9.60	Reduction (survey)
12	Liu 2008 [27]	48	8.00	Generalized RS
13	Qian+ 2008 [40]	48	8.00	Measures
14	Yang+ 2008 [55]	45	7.50	Dominance RS
15	Feng+ 2010 [11]	44	11.00	Soft sets
16	Yao YY 2010 [68]	41	10.25	Introduced three-way decision
17	Xiao+ 2009 [54]	41	8.20	App - forecasting
18	Bai+ 2010 [3]	38	9.50	Combining with grey system
19	Li+ 2008 [25]	38	6.33	App - prediction
20	Feng+ 2011 [12]	37	12.33	Soft sets

JingTao Yao and Yan Zhang, A Scientometrics Study of Rough Sets in Three Decades, RSKT 2013

- There are many new and young researchers, many of them from China, contributed to the highly cited papers in recent five year.
- Most of highly cited papers in last five years are extensions and applications of existing research.
- There is a need for new ideas and development.
- **The theory of three-way decisions, motivated by rough set three-regions but goes beyond rough sets, is a promising research direction that may lead to new breakthrough.**

Rough set approximations



Positive region

Boundary region

Negative region

Concept X

Two Formulations of Pawlak Rough Sets

- Rough sets as a pair of lower and upper approximations:

$$\begin{aligned} \underline{apr}(A) &= \{x \in U \mid [x] \subseteq A\}, \\ \overline{apr}(A) &= \{x \in U \mid [x] \cap A \neq \emptyset\}. \end{aligned}$$

- Rough sets as three-regions:

$$\begin{aligned} \mathrm{POS}(A) &= \{x \in U \mid [x] \subseteq A\}; \\ \mathrm{NEG}(A) &= \{x \in U \mid [x] \subseteq A^c\}; \\ \mathrm{BND}(A) &= (\mathrm{POS}(A) \cup \mathrm{NEG}(A))^c \\ &= \{x \in U \mid \neg([x] \subseteq A^c) \wedge \neg([x] \subseteq A)\}. \end{aligned}$$

- Z. Pawlak, Rough Sets, Theoretical Aspects of Reasoning about Data, Kluwer Academic Publishers, Dordrecht, 1991.

Decision-theoretic Rough Sets

- Consider a pair of thresholds (α, β) with $0 \leq \beta < \alpha \leq 1$
- Probabilistic rough sets as a pair of lower and upper approximations:

$$\begin{aligned} \underline{apr}_{(\alpha,\beta)}(A) &= \{x \in U \mid Pr(A|[x]) \geq \alpha\}, \\ \overline{apr}_{(\alpha,\beta)}(A) &= \{x \in U \mid Pr(A|[x]) > \beta\}. \end{aligned}$$

- Probabilistic rough sets as three-regions:

$$\begin{aligned} \mathrm{POS}_{(\alpha,\beta)}(A) &= \{x \in U \mid Pr(A|[x]) \geq \alpha\}, \\ \mathrm{NEG}_{(\alpha,\beta)}(A) &= \{x \in U \mid Pr(A|[x]) \leq \beta\}, \\ \mathrm{BND}_{(\alpha,\beta)}(A) &= \{x \in U \mid \beta < Pr(A|[x]) < \alpha\}. \end{aligned}$$

Three-way Decisions with Rough Sets

- Interpretation of three-way decisions:
  - Positive region: acceptance, rules of acceptance
  - Negative region: rejection, rules of rejection
  - Boundary region: noncommitment, further information/evidence is required.
- Pawlak rough sets vs. decision-theoretic rough sets
  - Can be uniformly interpreted in terms of three-way decisions.
  - Pawlak rough set model is a qualitative model:
    no incorrect acceptance error,
    no incorrect rejection error.
  - Decision-theoretic rough set model is a quantitative model:
    tolerance of incorrect acceptance error ($\leq 1-\alpha$),
    tolerance of incorrect rejection error ($\leq \beta$).

A Basic Question:

- Can we generalize three-way decisions into a larger context?
- We will try to answer this question today.

A Wide Range of Applications of Three-way Decisions

- Medical decision-making:
  Treatment, non-treatment, further investigation/testing
- Lurie, J.D., Sox, H.C.: Principles of medical decision making. Spine 24, 493-498 (1999)
- Pauker,, S.G., Kassirer, J.P.: The threshold approach to clinical decision making. The New England Journal of Medicine 302, 1109-1117 (1980)
- Schechter, C.B.: Sequential analysis in a Bayesian model of diastolic blood pressure measurement. Medical Decision Making 8, 191-196 (1988)

A Wide Range of Applications of Three-way Decisions

- Social judgement theory:
  Acceptance, rejection, and noncommitment
- Sherif, M., Hovland, C.I.: Social Judgment: Assimilation and Contrast Effects in Communication and Attitude Change. Yale University Press, New Haven (1961)

A Wide Range of Applications of Three-way Decisions

- Hypothesis testing in statistics:
  Accept, reject, and further test
- Wald, A.: Sequential tests of statistical hypotheses. The Annals of Mathematical Statistics 16, 117-186 (1945)

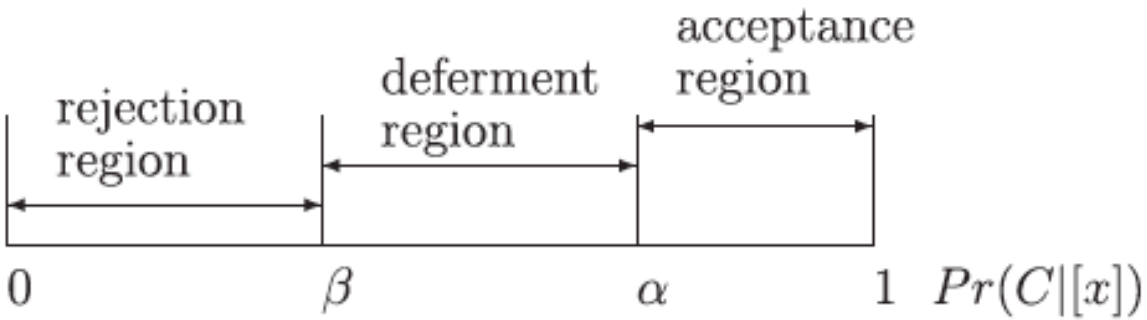
A Wide Range of Applications of Three-way Decisions

- Management sciences:
  Continue a program, stop a program, and further observation
- Goudey, R.: Do statistical inferences allowing three alternative decision give better feedback for environmentally precautionary decision-making. Journal of Environ- mental Management 85, 338-344 (2007)
- Woodward, P.W., Naylor, J.C.: An application of Bayesian methods in SPC. The Statistician 42, 461-469 (1993)

A Wide Range of Applications of Three-way Decisions

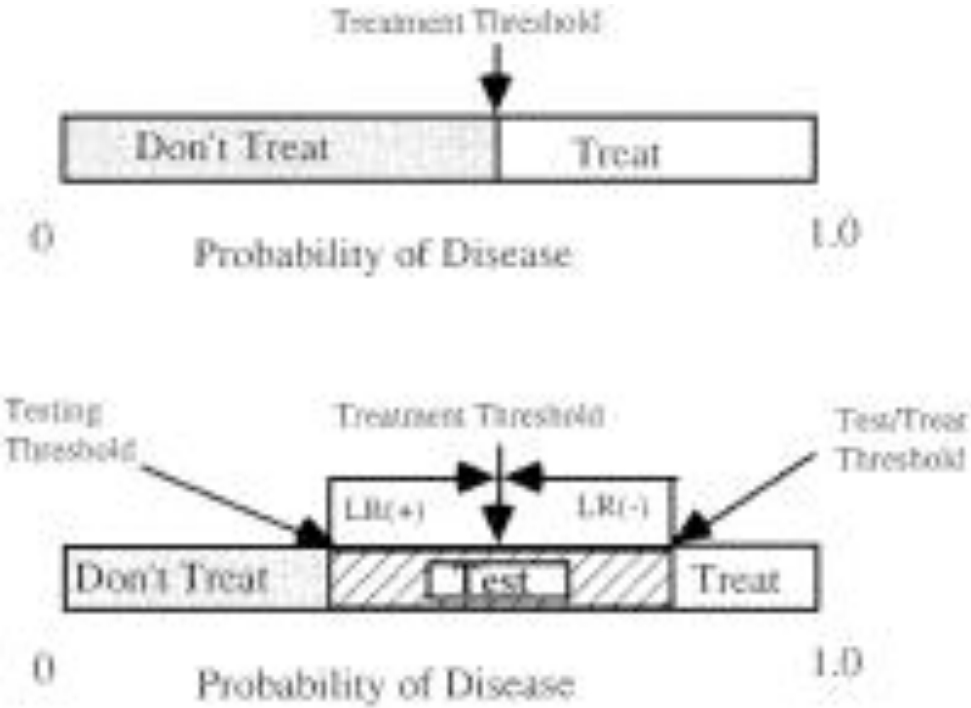
- Peering review process:
  Accept, reject, and further review
- Weller, A.C.: Editorial Peer Review: Its Strengths and Weaknesses. Information Today, Inc., Medford, NJ (2001)

Decision-theoretic Rough Sets



Yao, Y.Y., The superiority of three-way decisions in probabilistic rough set models. Information Sciences 181 (2011) 1080-1096.

Clinical three-way decisions



Lurie, J.D., Sox, H.C.: Principles of medical decision making. Spine 24, 493-498 (1999).

Observations

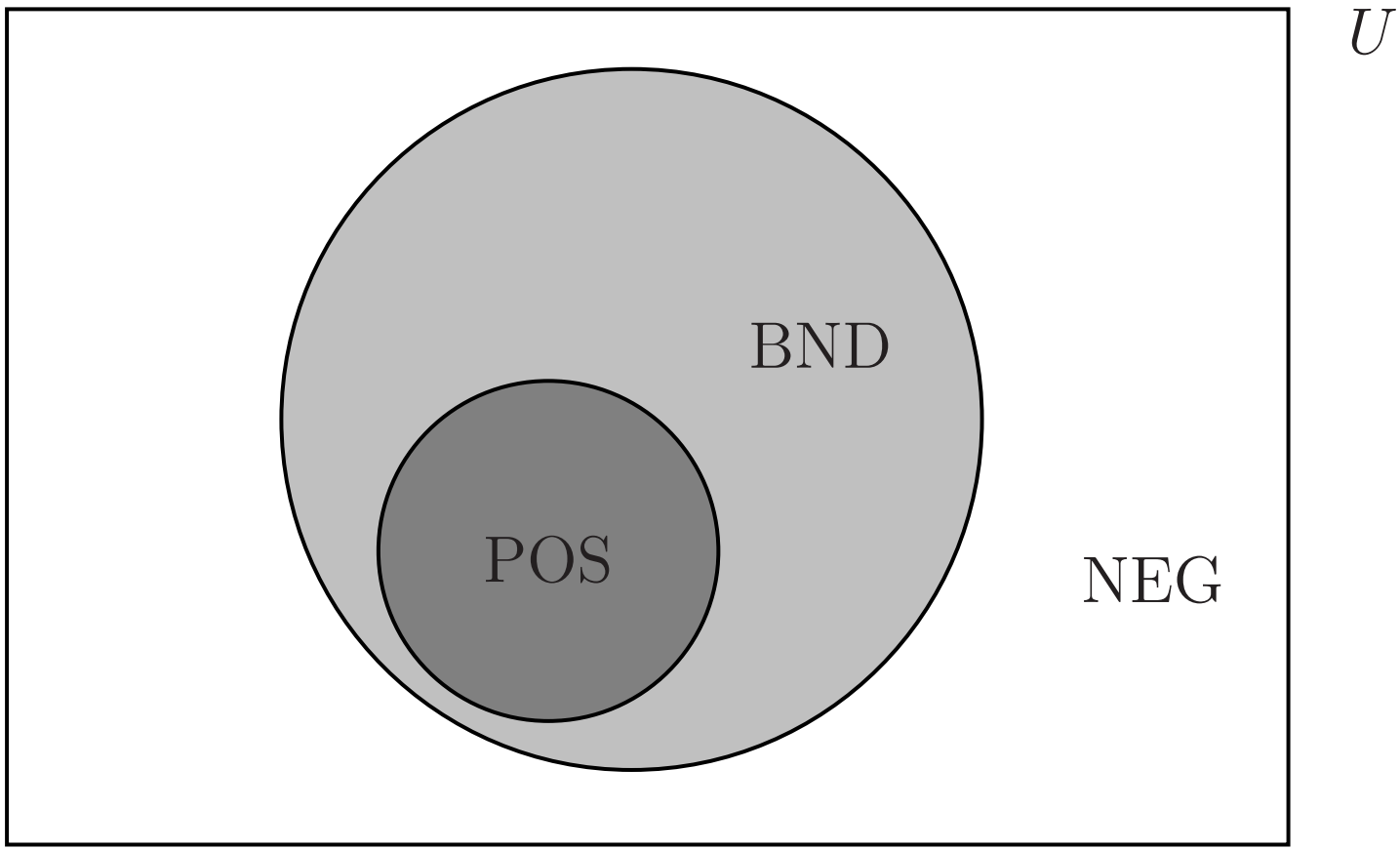
- Three-way decisions are everywhere.
- Three-way decisions are made everyday.
- Three-way decisions are normally described by using different vocabularies in different disciplines.

- There is still a lack of common framework or general theory on three-way decisions. (Additional information is welcome.)
- **It is a good time to start working on a theory of three-way decisions.**

三支决策的基本思想

The problem of three-way decisions.
设U是有限、非空实体集，C是有限条件集。三支决策是基于条件集C，将实体集U划分为三个两两不相交的区域（即正域、负域和边界域)。

三支决策的基本思想



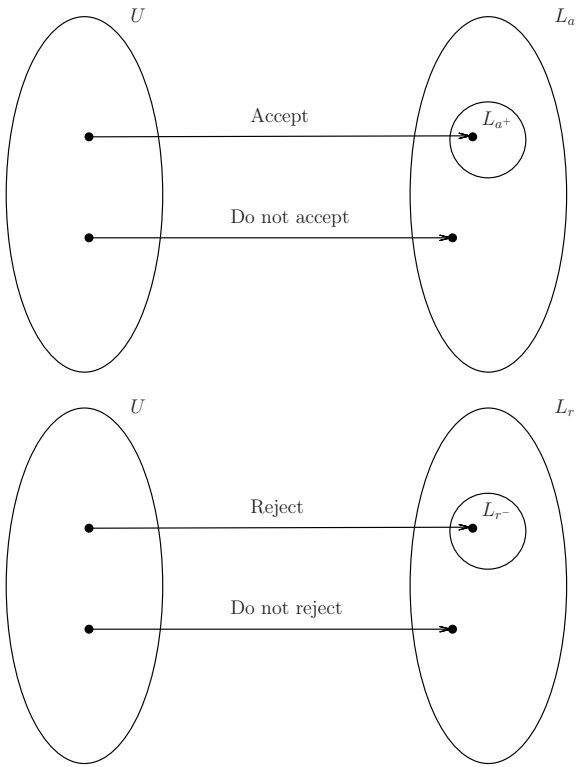
Basic ideas

- Build evaluation functions for dividing objects into three regions, corresponding to three decisions/actions.
  - Determine and interpret the values of evaluation functions for three-way decisions, that is, designated values for acceptance and designated values for rejection.
  - Determine three regions based on evaluation status values.
- Utilize the three regions, that is, process the three regions.

Basic Ingredients and Issues

- Values of evaluation functions:
  Construction and interpretation of a set of values for measuring satisfiability and a set of values for measuring non-satisfiability.
- Evaluation functions:
  Construction and interpretation of evaluations.
- Designated values:
  Determination and interpretation of designated values for acceptance and designated values for rejection.

Model 1: Three-way Decisions with a Pair of Poset-based Evaluations



2

Model 1: Three-way Decisions with a Pair of Poset-based Evaluations

Suppose U is a finite nonempty set and $(L_a, \preceq_a)$ $(L_r, \preceq_r)$ are two posets. A pair of functions $v_a : U \longrightarrow L_a$ and $v_r : U \longrightarrow L_r$ is called an acceptance evaluation and a rejection evaluation, respectively. For $x \in U$, $v_a(x)$ and $v_r(x)$ are called the acceptance and rejection values of x, respectively.

Model 1: Three-way Decisions with a Pair of Poset-based Evaluations

Let $\emptyset \neq L_a^+ \subseteq L_a$ be a subset of L_a called the designated values for acceptance, and $\emptyset \neq L_r^- \subseteq L_r$ be a subset of L_r called the designated values for rejection. The positive, negative, and boundary regions of three-way decisions induced by (v_a, v_r) are defined by:

$$\begin{aligned}
\mathrm{POS}_{(L_a^+, L_r^-)}(v_a, v_r) &= \{x \in U \mid v_a(x) \in L_a^+ \wedge v_r(x) \notin L_r^-\},\\
\mathrm{NEG}_{(L_a^+, L_r^-)}(v_a, v_r) &= \{x \in U \mid v_a(x) \notin L_a^+ \wedge v_r(x) \in L_r^-\},\\
\mathrm{BND}_{(L_a^+, L_r^-)}(v_a, v_r) &= (\mathrm{POS}_{(L_a^+, L_r^-)}(v_a, v_r) \cup \mathrm{NEG}_{(L_a^+, L_r^-)}(v_a, v_r))^c\\
&= \{x \in U \mid (v_a(x) \notin L_a^+ \wedge v_r(x) \notin L_r^-) \vee\\
&\qquad (v_a(x) \in L_a^+ \wedge v_r(x) \in L_r^-)\}.
\end{aligned}$$

Model 1

$(A, \bar{A})$-model \ $(R, \bar{R})$-model	rejection	non-rejection
acceptance	non-commitment (contradiction)	acceptance
non-acceptance	rejection	non-commitment

Model 1 (Special case: total order)

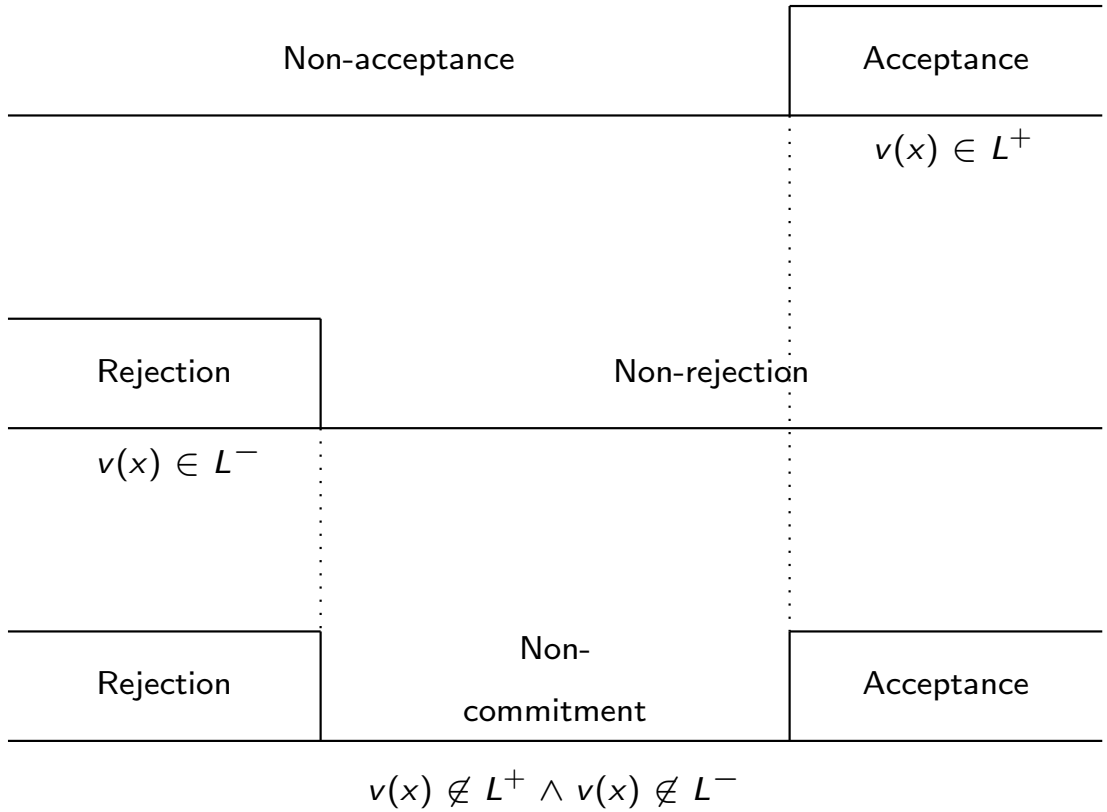
Two-way decisions for acceptance

+

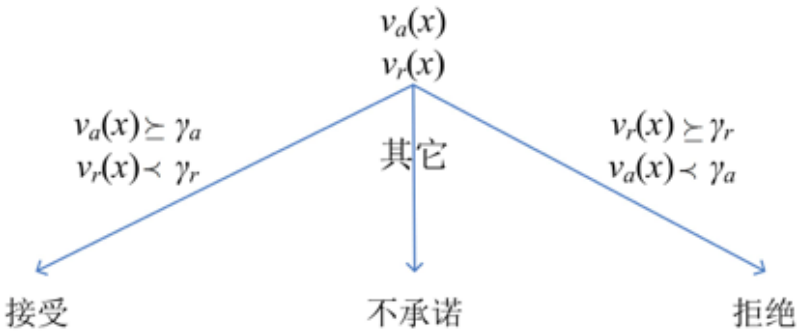
Two-way decisions for rejection

||

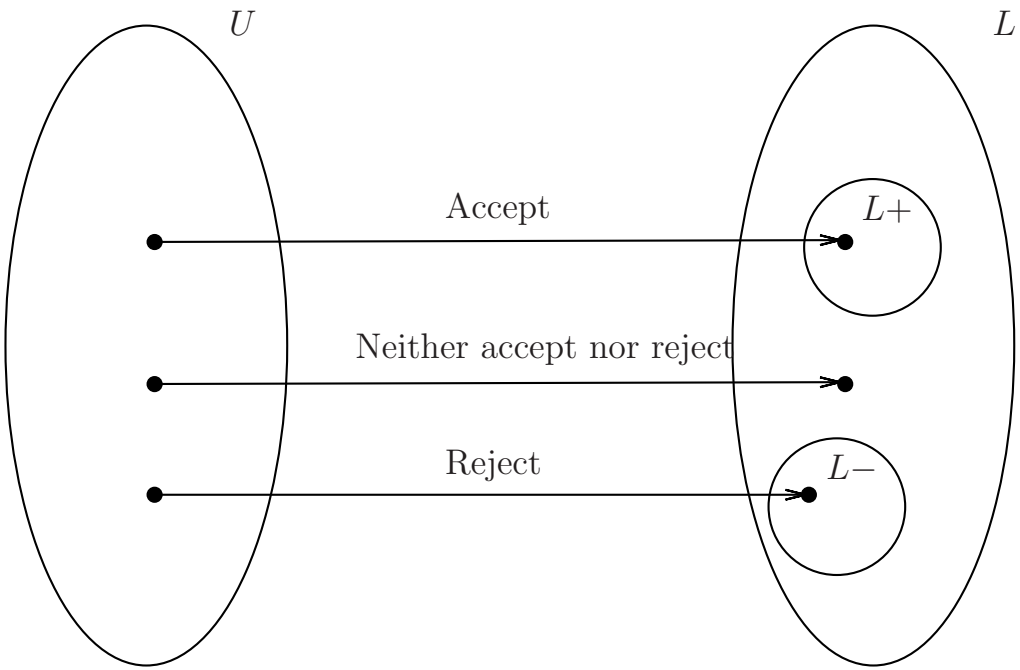
Three-way decisions



Model 1 (Special case: total order)



Model 2: Three-way Decisions with One Poset-based Evaluation

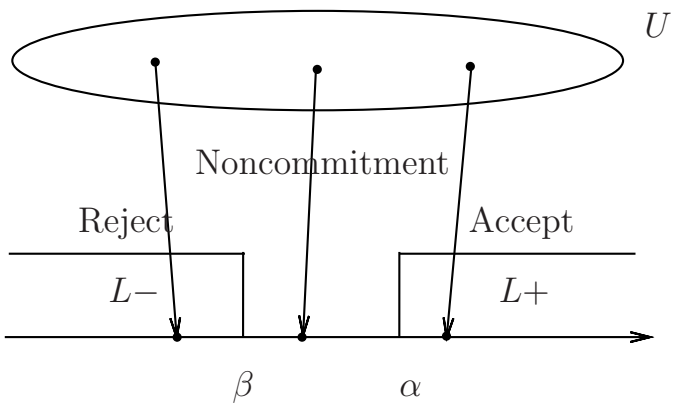


Model 2: Three-way Decisions with One Poset-based Evaluation

Suppose $(L, \preceq)$ is a poset. A function $v : U \longrightarrow L$ is called an acceptance-rejection evaluation. Let $L^+, L^- \subseteq L$ be two subsets of L with $L^+ \cap L^- = \emptyset$, called the designated values for acceptance and the designated values for rejection, rspectively. The positive, negative, and boundary regions of three-way decisions induced by v is defined by:

$$\begin{aligned}
\mathrm{POS}_{(L^+,L^-)}(v) &= \{x \in U \mid v(x) \in L^+\},\\
\mathrm{NEG}_{(L^+,L^-)}(v) &= \{x \in U \mid v(x) \in L^-\},\\
\mathrm{BND}_{(L^+,L^-)}(v) &= \{x \in U \mid v(x) \notin L^+ \wedge v(x) \notin L^-\}.
\end{aligned}$$

Model 3: Three-way Decisions with an Evaluation Using a Totally Ordered Set

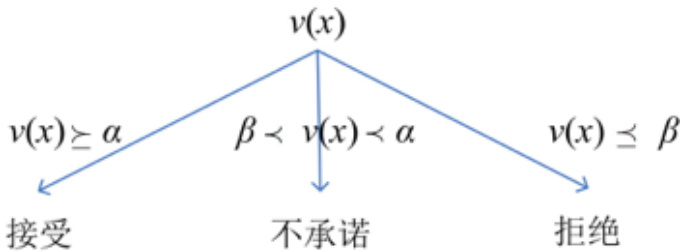


Model 3: Three-way Decisions with an Evaluation Using a Totally Ordered Set

Suppose $(L, \preceq)$ is a totally ordered set, that is, $\preceq$ is a total order. For two elements α, β with $\beta \prec \alpha$ (i.e., $\beta \preceq \alpha \wedge \neg(\alpha \preceq \beta)$), suppose that the set of designated values for acceptance is given by $L^+ = \{t \in L \mid t \succeq \alpha\}$ and the set of designated values for rejection is given by $L^- = \{b \in L \mid b \preceq \beta\}$. For an evaluation function $v : U \longrightarrow L$, its three regions are defined by:

$$\begin{aligned} \mathrm{POS}_{(\alpha,\beta)}(v) &= \{x \in U \mid v(x) \succeq \alpha\}, \\ \mathrm{NEG}_{(\alpha,\beta)}(v) &= \{x \in U \mid v(x) \preceq \beta\}, \\ \mathrm{BND}_{(\alpha,\beta)}(v) &= \{x \in U \mid \beta \prec v(x) \prec \alpha\}. \end{aligned}$$

Model 3



Construction of Evaluations (An Example)

Suppose $C = \{c_1, c_2, \ldots, c_m\}$ are a set of m criteria. Suppose $v_{c_i} : U \longrightarrow \Re$ denotes an evaluation based on criterion v_i, $1 \leq i \leq m$. An overall evaluation function $v : U \longrightarrow \Re$ may be simply defined by a linear combination of individual evaluations:

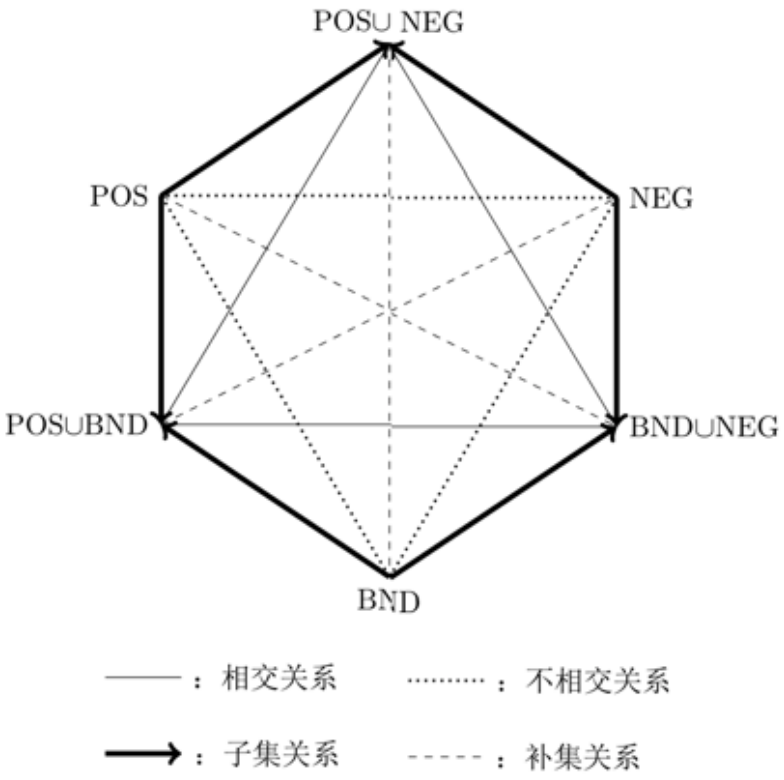
$$v(x) = v_{c_1}(x) + v_{c_2}(x) + \ldots + v_{c_m}(x).$$

Determination of Designated Sets (An Example)

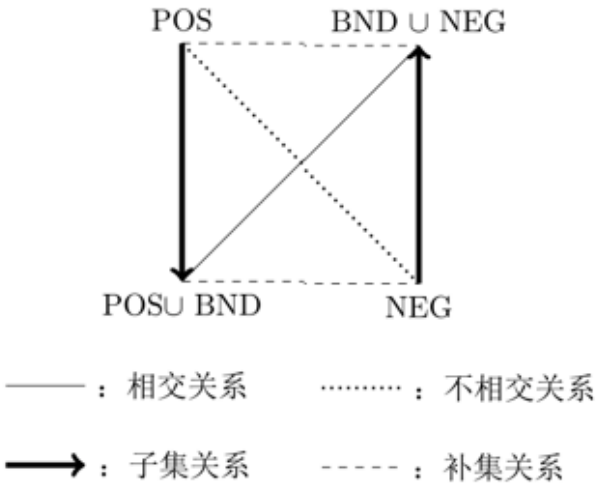
Let $R_P(\alpha, \beta)$, $R_N(\alpha, \beta)$ and $R_B(\alpha, \beta)$ denote the risks of the positive, negative, and boundary regions, respectively. It is reasonable to require that the sets of designated values are chosen to minimize the following overall risks:

$$R(\alpha, \beta) = R_P(\alpha, \beta) + R_N(\alpha, \beta) + R_B(\alpha, \beta).$$

三支决策的几何描述



三支决策的几何描述



Interval Sets and Three-valued Logic

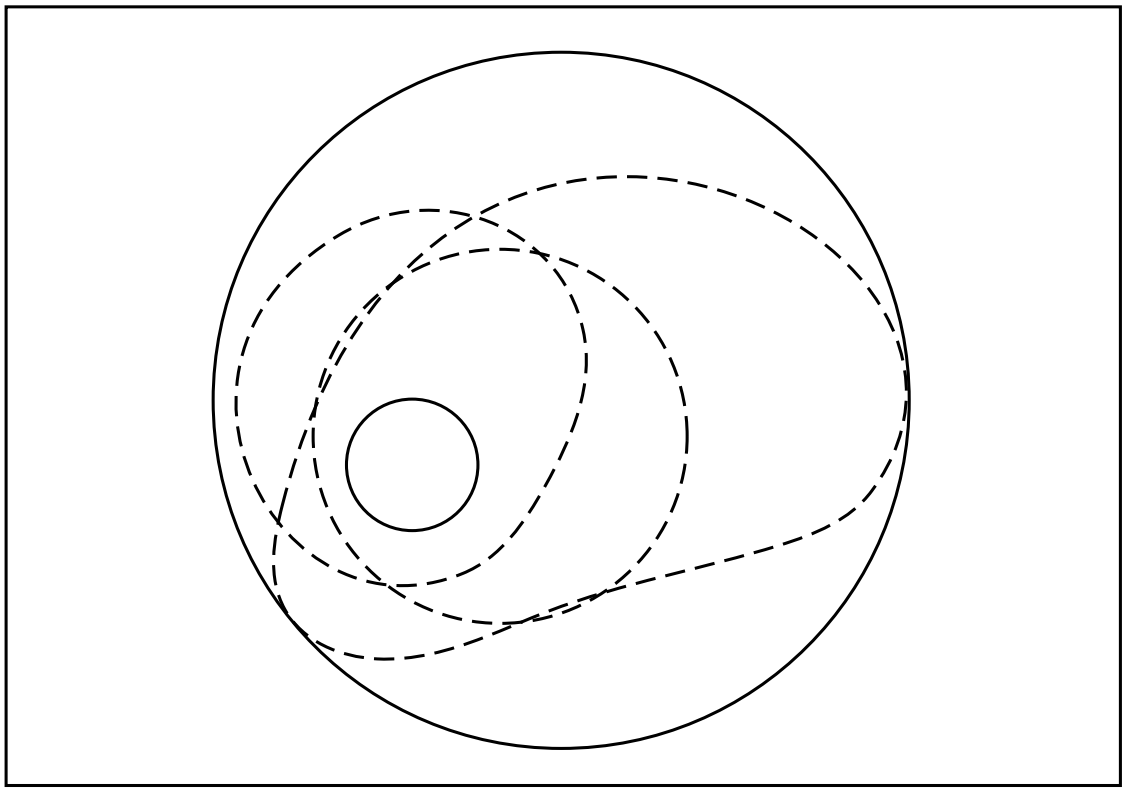
- A closed interval set is a subset of 2^U of the form,

$$[A_l, A_u] = \{A \in 2^U \mid A_l \subseteq A \subseteq A_u\},$$

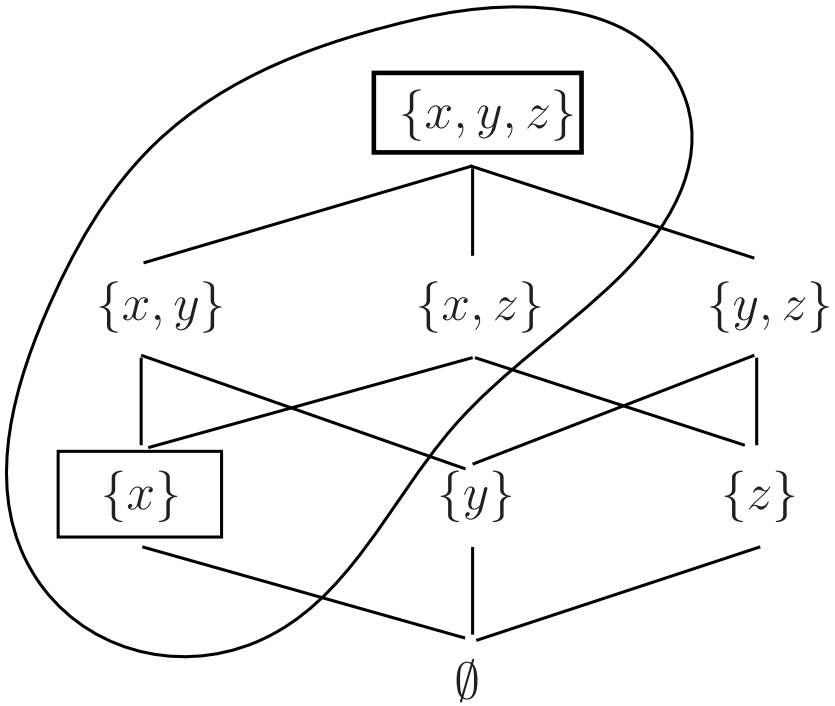
  where it is assumed that $A_l \subseteq A_u$, and A_l and A_u are called the lower and upper bound, respectively.

- Yao, Y.Y., Interval-set algebra for qualitative knowledge representation. In: Proceedings of the 5th International Conference on Computing and Information, pp. 370-374 (1993)

Interval Sets and Three-valued Logic



Interval Sets and Three-valued Logic



$$[\{x\}, \{x, y, z\}] = \{\{x\}, \{x, y\}, \{x, z\}, \{x, y, z\}\}.$$

Interval Sets and Three-valued Logic

- Three Regions of an interval sets: positive regions, boundary regions, and negative regions,

$$\begin{aligned}
\mathrm{POS}([A_l, A_u]) &= A_l, \\
\mathrm{NEG}([A_l, A_u]) &= (A_u)^c \\
\mathrm{BND}([A_l, A_u]) &= A_u - A_l \\
&= (\mathrm{POS}([A_l, A_u]) \cup \mathrm{BND}([A_l, A_u]))^c.
\end{aligned}$$

Interval Sets and Three-valued Logic (Model 3)

- The sets of designated values for acceptance and rejection are defined by a pair of thresholds (T, F), namely,
  $L^+ = \{a \in L \mid T \preceq a\} = \{T\}$ and
  $L^- = \{b \in L \mid b \preceq F\} = \{F\}$.
- Three-way decisions:

$$\begin{aligned}
\mathrm{POS}_{(T,F)}([A_l, A_u]) &= \{x \in U \mid v_{[A_l,A_u]}(x) \succeq T\} = A_l, \\
\mathrm{NEG}_{(T,F)}([A_l, A_u]) &= \{x \in U \mid v_{[A_l,A_u]}(x) \preceq F\} = (A_u)^c, \\
\mathrm{BND}_{(T,F)}([A_l, A_u]) &= \{x \in U \mid F \prec v_{[A_l,A_u]}(x) \prec T\} = A_u - A_l.
\end{aligned}$$

Three-way approximations of a fuzzy sets

L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338-353.

- 模糊集

$$\mu_A : U \longrightarrow [0, 1]$$

- "... one can introduce two levels α and β $(0 < \alpha < 1, 0 < \beta < 1, \alpha > \beta)$ and agree to say that (1) 'x belongs to A' if $f_A(x) \geq \alpha$; (2) 'x does not belong to A' if $f_A(x) \leq \beta$; and (3) 'x has an indeterminate status relative to A' if $\beta < f_A(x) < \alpha$. This leads to a three-valued logic (Kleene, 1952) with three truth values: T $(f_A(x) \geq \alpha)$, F $(f_A(x) \leq \beta)$, and U $(\beta < f_A(x) < \alpha)$."

Three-way approximations of a fuzzy sets

- 模糊集

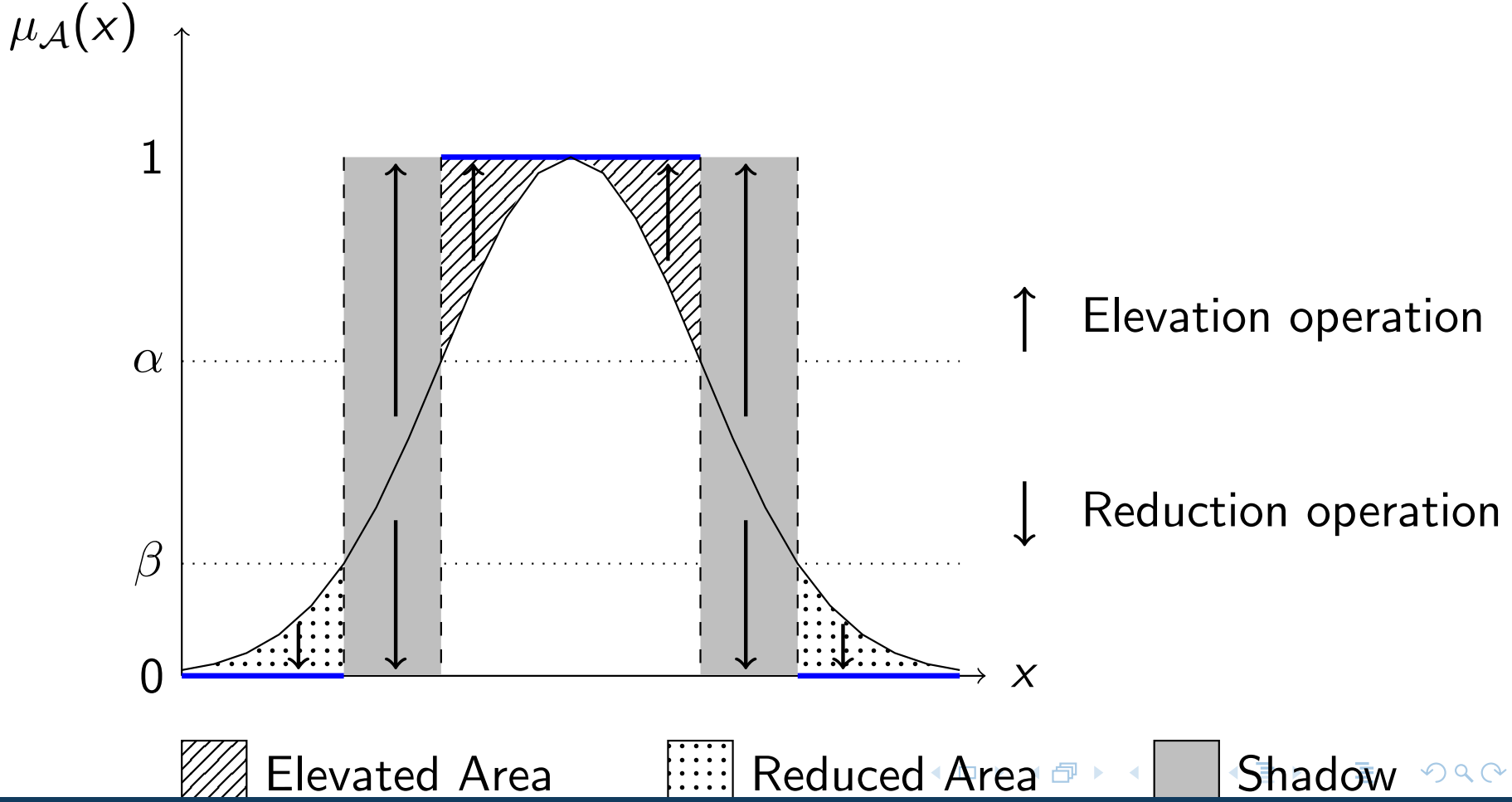
$$\mu_A : U \longrightarrow [0, 1]$$

- The three regions are defined respectively by:

$$\begin{aligned}
\mathrm{POS}_{(\alpha,\beta)}(\mu_A) &= \{x \in U \mid \mu_A(x) \geq \alpha\}, \\
\mathrm{NEG}_{(\alpha,\beta)}(\mu_A) &= \{x \in U \mid \mu_A(x) \leq \beta\}, \\
\mathrm{BND}_{(\alpha,\beta)}(\mu_A) &= \{x \in U \mid \beta < \mu_A(x) < \alpha\}.
\end{aligned}$$

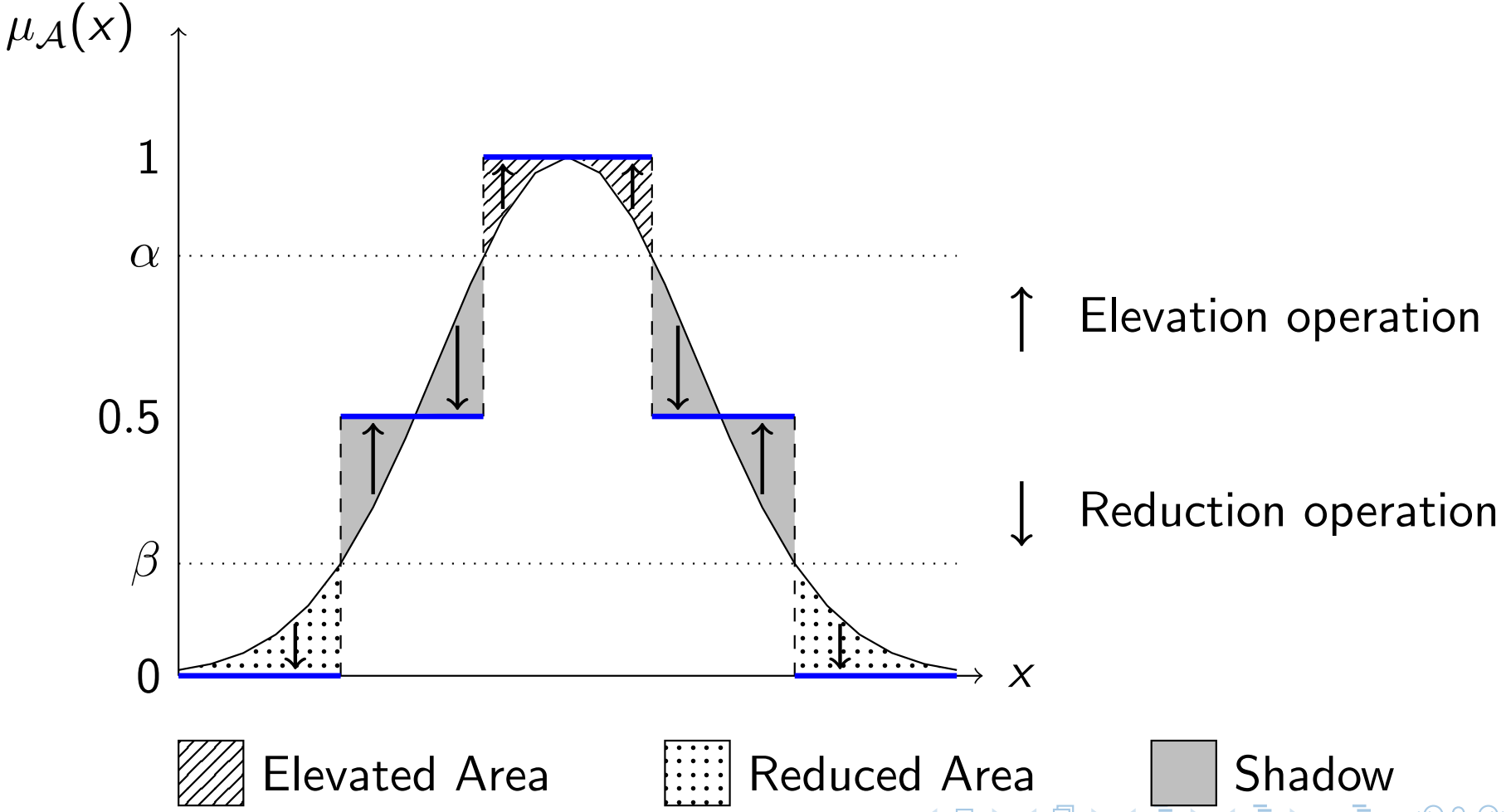
Shadowed Sets

W. Pedrycz, Shadowed sets: Representing and processing fuzzy sets, IEEE Tran. on System, Man and Cybernetics 28 (1998) 103-109.



Three-way approximations of fuzzy sets

Deng, X.F., and Yao, Y.Y., Decision-theoretic three-way approximations of fuzzy sets, Information Sciences, 2014.



Conclusion

- 三支决策的研究是一种新的尝试，欲将众多领域中所用到的决策方法和思想同人类直观的信息处理模式联系在一起，以期提供一个普适的理论。
- 三支决策的研究虽然刚刚开始，却已取得了很多令人鼓舞的成果。
- 许多学者的关注和欣然加入，更为该理论的发展注入了新鲜血液、打下了坚实基础。

Recommendation

- 从对一个现有理论的小修补到一个新理论提出和发展，这是在研究理念上的一次飞跃。

Recommendation

- 三支决策的进一步研究应强调多样性和创新性。
- 跳出粗糙集的圈子。
- 我们共同的努力将可能建立起一个完善的三支决策理论。

Thank you for your attention

Thanks!

谢谢!

For more information, see
http://www.cs.uregina.ca/∼yyao